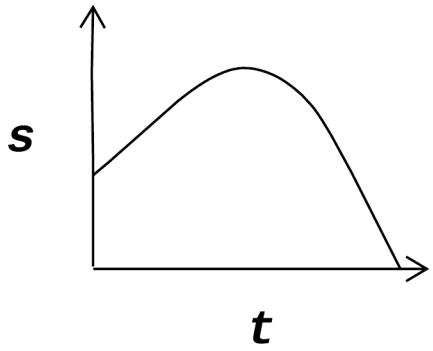
Introduction to Derivatives for Physics Students

Let's start with an object that is thrown up from 5 m above the ground at a speed of 10 m/s. We know gravity will pull the object down at 9.8 m/s/s. Because we are taking distance above the ground to be positive, gravity will be negative. We can express the objects height above the ground as a function of time:

$$\mathrm{s} = 5\ \mathrm{m} + \ 10\ \mathrm{m/s}\ + \frac{1}{2}(-9.8\ \mathrm{m/s/s})\ t^2$$

Both the physics student and the calculus student will recognize this function is quadratic and the parabola will be concave down. They would agree the sketch of the function will look like this:



The calculus student would be able to find the first and second derivatives of the function, and she would use this notation:

$s(t)$ for the function

$s'(t)$ or $\frac{ds}{dt}$ for the first derivative

$s''(t)$ or $\frac{d^2 s}{d\,t^2}$ for the second derivative

We have checked the units of the function, they are all in MKS, so we know the units 1 time are seconds, so can ignore the units as we continue with the math and when we can make sure to note the units when we find our final answer. We can also rewrite the function to reflect the typical notation:

$$s = -4.9\,t^2 + 10\,t + 5$$

This tells us that at time t seconds, the height of the object above the ground is s meters.

The instantaneous velocity (the rate at which the distance is changing) of the object at time t seconds is given by the first derivative of the function. Because the function is quadratic, we can follow a simple rule (which is called the Power Rule) when "differentiating the equation."

$$\text{If } s(t) = x^r, \text{ then the first derivative is } s' = r(x)^{r-1}$$

The Power Rule is applied to each term that is expressed as a function of t; also, coefficients remain in the equation, so the equation of the first derivative of our function is:

$$s'(t) = -4.9(2)t + 10 \text{ or}$$

$$s' = -9.8t + 10$$

This tells us that at time t seconds, the speed of the object above the ground is s' meters per second.

The instantaneous acceleration (the rate at which the velocity is changing) of the object at time t seconds is found by applying the Power Rule to the equation of the first derivative. In this case, $s'(t)$ is linear (in the form $y = mx + b$), so

$$s''(t) = -9.8$$

While this may seem a lot of work to find values that are expressed in the problem, this physics problem can help the calculus student understand why we find derivatives. These equations tell us how values are changing.